DO NOW

If an arithmetic sequence is defined recursively as $a_1 = 5$ and $a_n = a_{n-1} + 4$,

a. Find the common difference.
b. Write the general rule and simplify to the explicit formula.
c. Find the 15th term.

a. $d = 4$

b. $a_n = a_1 + (n-1)d$
$a_n = 5 + (n-1)(4)$
$a_n = 5 + 4n - 4$
$\boxed{a_n = 4n+1}$

c. $a_{15} = 4(15)+1$
$a_{15} = 60+1$
$\boxed{a_{15} = 61}$

9.5 Geometric Sequences

Geometric sequence - multiply by a common ratio, r, to find the next number.

*EXPONENTIAL FUNCTION

Example: 2, 6, 18, 54, 162...
×3 ×3 ×3

Find r and the next two terms.

1. 2, 8, 32, 128... $r=4$
   512, 2048
2. -1, -3, -9, -27... $r=3$
   -81, -243
3. -3, 12, -48, 192... $r=-4$
   -768, 3072
4. 3, 15, 75, 375... $r=5$
   1875, 9375

Write the first 5 terms of a geometric sequence as defined.

5. $a_1 = 2, r = 5$
   2, 10, 50, 250, 1250
6. $a_1 = -3, r = 2$
   -3, -6, -12, -24, -48
7. $a_1 = 8, r = \frac{1}{2}$
   8, 4, 2, 1, $\frac{1}{2}$
8. $a_1 = 4, r = -3$
   4, -12, 36, -108, 324

Writing a geometric sequence recursively:

$a_n = a_{n-1} \cdot r$
$a_n \leftarrow$ general term
$a_{n-1} \leftarrow$ term before a_n
$r \leftarrow$ common ratio

Example: Given 2, 10, 50, 250...
$r=5$
$a_n = a_{n-1} \cdot r$
$a_n = a_{n-1} \cdot 5$
$\boxed{a_n = 5a_{n-1}}$

Writing a geometric sequence explicitly (General Rule):

$a_n = a_1 \cdot r^{n-1}$
$a_n \leftarrow$ general term
$a_1 \leftarrow$ first term
$r \leftarrow$ common ratio (base)
$n \leftarrow$ term #
$n-1 \leftarrow$ exponent

Example: Given 2, 10, 50, 250...
$r=5$
$a_n = a_1 \cdot r^{n-1}$
$\boxed{a_n = 2 \cdot 5^{n-1}}$
or
$a_n = 2(5^{n-1})$

9. Consider the geometric sequence: -3, -15, -75, -375.
   a. Find r, the common ratio.
   b. Find the explicit formula and simplify.
   c. Find the 10th term in the sequence.

a. $r=5$

b. $a_n = a_1 \cdot r^{n-1}$
$\boxed{a_n = -3 \cdot 5^{n-1}}$

c. $a_{10} = -3 \cdot 5^{10-1}$
$a_{10} = -3 \cdot 5^9$
$a_{10} = -3 \cdot 1{,}953{,}125$
$\boxed{a_{10} = -5{,}859{,}375}$

10. If a geometric sequence is defined recursively as $a_1 = 6$ and $a_n = 3a_{n-1}$
   a. Find the common ratio.
   b. Write the general rule and simplify to the explicit formula.
   c. Find the 8th term.

a. $r=3$

b. $a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 6 \cdot 3^{n-1}}$

c. $a_8 = 6 \cdot 3^{8-1}$

$a_8 = 6 \cdot 3^7$

$a_8 = 6 \cdot 2187$

$\boxed{a_8 = 13{,}122}$

11. In the geometric sequence -7, x_2, x_3, 189...
   a. Find r, the common ratio.
   b. Find the missing terms x_2 and x_3.
   c. Find the 5th term.

$a_1 = -7$
$a_4 = 189$

a. $a_n = a_1 \cdot r^{n-1}$

use $a_1 = -7$
$a_4 = 189$

$189 = -7 \cdot r^{4-1}$

$\frac{189}{-7} = r^3$

$-27 = r^3$

$\sqrt[3]{-27} = r$

$\boxed{-3 = r}$

b. $x_2 = 21$

$x_3 = -63$

c. $a_n = a_1 \cdot r^{n-1}$

$a_n = -7 \cdot (-3)^{n-1}$

$a_5 = -7 \cdot (-3)^{5-1}$

$a_5 = -7 \cdot (-3)^4$

$a_5 = -7 \cdot 81$

$\boxed{a_5 = -567}$

12. On the first swing, the length of the arc through which a pendulum swings is 24 inches. The length of each successive swing is $\frac{7}{8}$ of the preceding swing. Find the length of the arc on the fifth swing. (Round your answer to the nearest tenth).

$a_1 = 24$

$r = \frac{7}{8}$

Find a_5

$a_n = a_1 \cdot r^{n-1}$

$a_n = 24 \cdot \left(\frac{7}{8}\right)^{n-1}$

$a_5 = 24 \cdot \left(\frac{7}{8}\right)^{5-1}$

$a_5 = 24 \cdot \left(\frac{7}{8}\right)^4$

$\boxed{14.1 \text{ inches}}$

HOMEWORK

Worksheet - HW 9.5